Math 53, Discussion 105

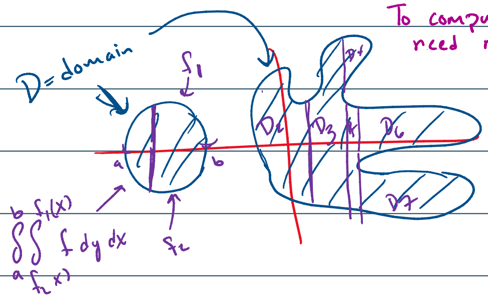
Wednesday, March 18, 2020 9:09 AM

- 15.3 (integral over arbitrary domain
- 15.4 (double integral in polar coordinates)

15.3 $f(x,y)$ = function of two variables

want to find volume under curve of f in some region

To compute $\int_D f\,dA$ need many integrals



Ex $f(x,y) = e^x \sin(y)$

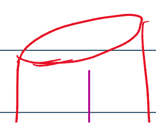
15.4 problems:

1. $\int_D (x+y)dA$ when D is bouded by $y=\sqrt{x}$ and $y=x^2$

Answer: possibly $(10/3)^{-1}$

1. Set up an integral to evaluate the volume of a cylinder of radius r and height h
2. Set up an integral to evaluate the volume of a sphere of radius r
3. Find the volume enclosed by the parabolic

1) let $f(x,y)=h$

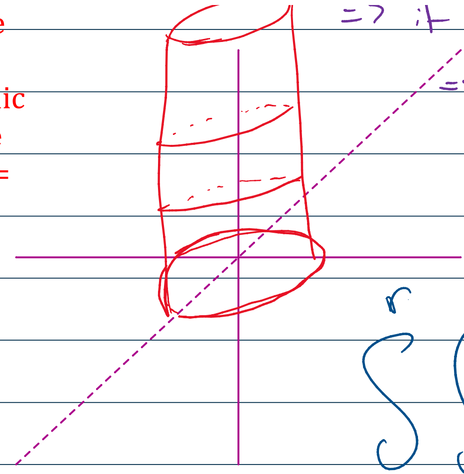


$\Rightarrow$ if D = circle radius r

$\Rightarrow \iint f\,dA =$

$\sqrt{r^2-x^2}$

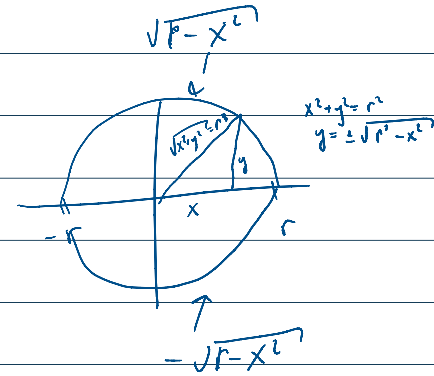
2. Set up an integral to evaluate the volume of a sphere of radius r
3. Find the volume enclosed by the parabolic cylinders $y = 1 - x^2, y = x^2 - 1$ and the planes $x + y + z = 2, 2x + 2y - z + 10 = 0$



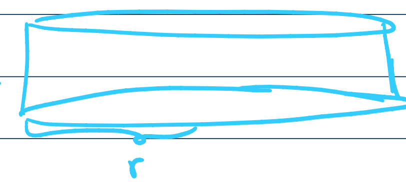
=> if D = circle radius r

$\Rightarrow \iint_D f\,dA$ = volume under f = volume of cylinder.

$$\int_{-r}^{r}\int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} h\,dy\,dx$$



How to remember r



15.4: polar integration

Formula: $\int\int f(r,\theta)\, r\, dr\, d\theta$

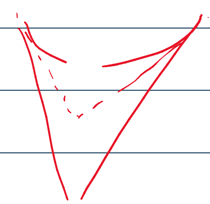
odd

Area of circle $= \pi r^2 = \iint_{\text{circle}} 1\,dA$ = use polar coordinates

$$= \int_0^{2\pi}\int_0^{r} 1 \cdot r\,dr\,d\theta$$

$$\int_0^{2\pi} \frac{r^2}{2}\Big|^{r} d\theta$$

1. Compute $\iint_D xy\,dA$ with D the disk with center at the origin and radius 3



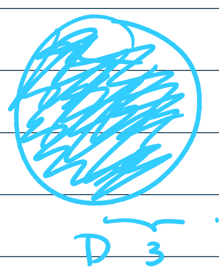
need to convert (x,y) to polar.

$x = r\cos\theta \quad y = r\sin\theta$

$$\int_0^{2\pi}\int_0^{3} r\cos\theta\, r\sin\theta\, r\,dr\,d\theta$$

$$\int_0^{2\pi} \cos\theta\sin\theta\, \frac{r^4}{4}\Big|_0^3 d\theta$$

$\frac{\sin(2\theta)}{2}$



$\int_0^{2\pi} \cos\theta \sin\theta \, \frac{r^4}{4}\Big|_0 \, d\theta$

$\frac{\sin(2\theta)}{2}$

$\frac{81}{4}\int_0^{2\pi} \cos\theta \sin\theta \, d\theta$

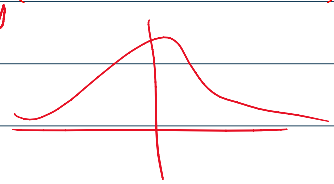
$\frac{\sin^2\theta}{2}\Big|^{2\pi} = 0$ because f is a saddle point & get symmetry

$\frac{1}{2}\int_{\theta_1}^{\theta_2} r^2 \, d\theta$

we can get by our new double integral formula

$= \frac{2\pi}{2} r^2 = \pi r^2$

1. Compute the area of one loop of the rose $r = \cos 3\theta$
2. Compute the area of a sphere of radius a
3. Fun: learn the trick how to compute $\int_{\infty}^{\infty} e^{-x^2}\,dx$ (area under a Gaussian)



Before can compute

now